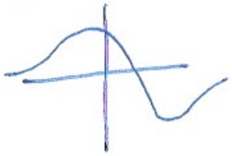
10.1 PARAMETRIC CURVES

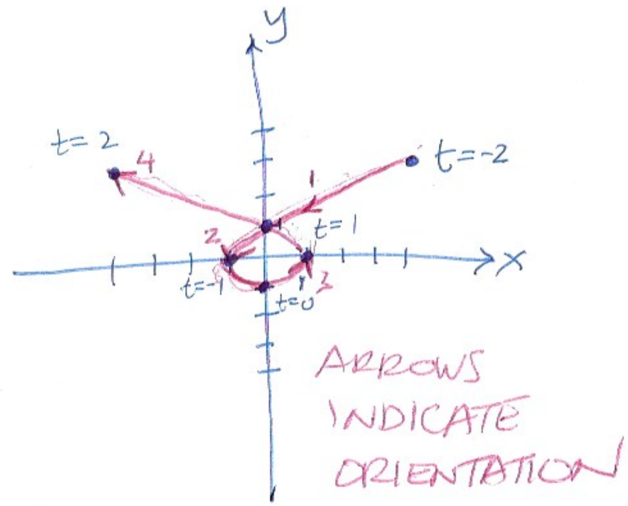
$y=f(x)$ $\qquad$ $x^2+y^2=1$



$$\left.\begin{array}{l} x=f(t) \\ y=g(t) \end{array}\right\}$$ PARAMETRIC EQUATIONS IN PARAMETER t

eg. $x=2t-t^3$ $\quad t\in[-2,2]$
$y=t^2-1$

t	x	y	(x,y)
-2	$-4-8=4$	3	$(4,3)$
-1	$-2-1=-1$	0	$(-1,0)$
0	$0-0=0$	-1	$(0,-1)$
1	$2-1=1$	0	$(1,0)$
2	$4-8=-4$	3	$(-4,3)$



FIND X-INT'S (IE. WHEN $y=0$)

$y=t^2-1=0 \longrightarrow t=\pm 1 \longrightarrow x=2-1=1$
$x=-2-1=-1$

FIND y-INT'S (IE. WHEN $x=0$)

$x=2t-t^3=0 \longrightarrow t(2-t^2)=0 \longrightarrow t=0,\sqrt{2},-\sqrt{2}$
$y=-1,1$

CONSIDER $x = t^2 - 3t$

$y = 2t - 1$

ELIMINATE THE PARAMETER

① BY SUBSTITUTION : SOLVE FOR t IN 1 EQ'N

SUBSTITUTE INTO THE OTHER EQ'N + SIMPLIFY

$$y = 2t - 1 \longrightarrow t = \tfrac{1}{2}(y+1)$$

$$x = \left(\tfrac{1}{2}(y+1)\right)^2 - 3\left(\tfrac{1}{2}(y+1)\right)$$

$$x = \tfrac{1}{4}y^2 + \tfrac{1}{2}y + \tfrac{1}{4} - \tfrac{3}{2}y - \tfrac{3}{2}$$

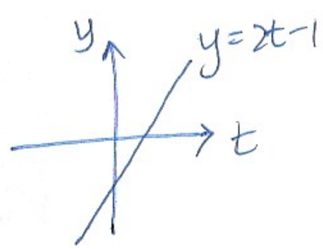
$$x = \tfrac{1}{4}y^2 - y - \tfrac{5}{4}$$

PARABOLA OPENS TO THE RIGHT

TO GRAPH,
FIND X-INT + Y-INT
+ SKETCH GRAPH

SO $x = t^2 - 3t$, $y = 2t - 1$ IS PART OF THAT PARABOLA

BUT WHICH PART + IN WHICH ORIENTATION ?



AS t GOES FROM $-\infty$ TO ∞

y GOES $-\infty$ TO ∞

CONSIDER $x = t^4 - 3t^2$

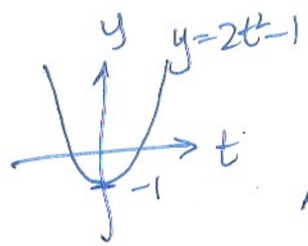
$y = 2t^2 - 1$

ELIMINATE THE PARAMETER

$$t^2 = \frac{1}{2}(y+1)$$

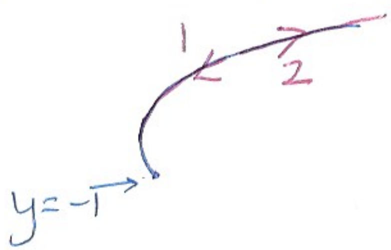
$$x = (t^2)^2 - 3t^2 = \left(\frac{1}{2}(y+1)\right)^2 - 3\left(\frac{1}{2}(y+1)\right)$$

$$x = \frac{1}{4}y^2 - y - \frac{5}{4}$$



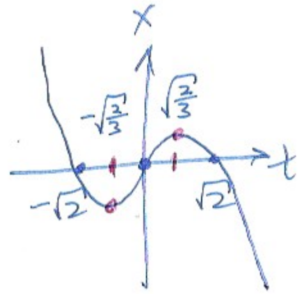
AS t GOES FROM $-\infty$ TO ∞

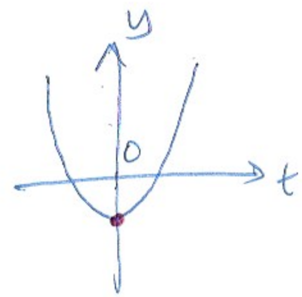
y $\quad\quad$ ∞ TO -1 TO ∞

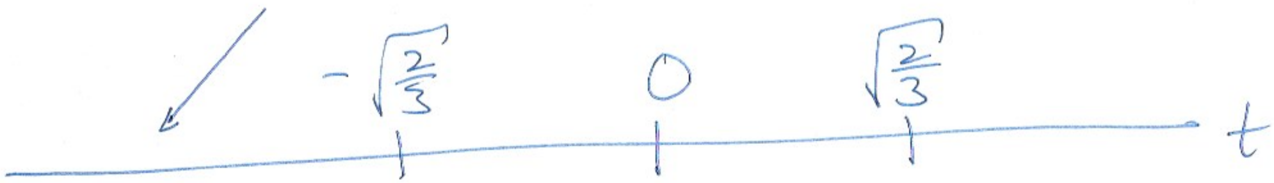


RECONSIDER $x = 2t - t^3 = t(2 - t^2)$ $\qquad x'(t) = 2 - 3t^2 = 0$

$y = t^2 - 1$ $\qquad t = \pm\sqrt{\frac{2}{3}}$







X DECR ←

y DECR ↓